

Surplus Angle and Sign-flipped Coulomb Force in Projectable Hořava-Lifshitz Gravity

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We obtain the static spherically symmetric vacuum solutions of Hořava-Lifshitz gravity theory, imposing the detailed balance condition only in the UV limit. We find the solutions in two different coordinate systems, the Painlevé-Gullstrand coordinates and the Poincaré coordinates, to examine the consequences of imposing the projectability condition. The solutions in two coordinate systems are distinct due to the non-relativistic nature of the HL gravity. In the Painlevé-Gullstrand coordinates compiling with the projectability condition, the solution involves an additional integration constant which yields surplus angle and implies attractive Coulomb force between same charges.

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I. INTRODUCTION

The UV complete theory of gravity has been a long-felt want since general relativity (GR) and quantum field theory were established. For past two decades, string theory has been considered to be a strongest and possibly unique candidate. Recently Hořava proposed a more practical/economical way to achieve this goal based on quantum field theory by giving up the general covariance of GR [1, 2]. In this theory, the time and the space show different scaling behaviors at the UV regime. It modifies the high momentum behavior of the graviton propagator and renders the theory power-counting renormalizable.

This novel feature of Hořava-Lifshitz (HL) gravity attracted much interest and various aspects of the theory have been investigated, including the properties and the consistency of the theory [3, 4, 6, 7], the properties of classical solutions including black holes and their thermodynamic properties [8, 9], the cosmological aspects [10, 11], the perturbation spectrum and the gravitational wave production [12], and the phenomenological sides [13]. One notable feature of the theory is that the general covariance (full 4D diffeomorphism) is reduced to the foliation-preserving diffeomorphism. Thus, we have additional degree of freedom, a dynamical scalar [6, 7], but with a first-order (in time) equation of motion. It gives rise to subtleties in canonical quantization, instability problem, and strong coupling problem [4]. On the other hand, the theory must be reduced to GR in the IR limit. It means that the symmetry must be enlarged at IR, the so-called emergent symmetry. Since HL gravity is supposed to be a quantum theory, it depends on the running of coupling constants, of which we do not have full understanding yet. This may lead to phenomenological difficulties, for example, the energy dependence in the limiting speed $c^2(E)$ and $\delta c^2(E)$. To get the better control over these problems, we may impose the projectability condition on the metric and the detailed balance conditions on the coupling constants of the theory. So far, it seems that imposing the detailed balance condition up to the IR regime is problematic and imposing the projectability is favorable.

Though the explicit quantization and the proof of the renormalizability are yet to be explored, it is interesting to see the consequences of such an approach at the IR regime, that is, in the classical solutions and to find the observable signatures through them. In this paper, we obtain the static spherically symmetric vacuum solutions of HL gravity imposing the detailed balance conditions only in the UV limit. Thus specific relations between parameters in the four or less spatial derivative terms are not imposed. For HL gravity, the choice of coordinates is important because the theory lacks the general covariance and is intrinsically non-relativistic. To define a theory, we must specify a frame where the action is defined. Without knowledge about the preferred frame a priori, we choose two coordinate systems, the Painlevé-Gullstrand (PG) coordinates and the Poincaré coordinates for comparison. The former coordinate system is better motivated for HL gravity since it satisfies the projectability condition [1]. The comparison of the solutions in two coordinate system will contrast the differences between projectable and non-projectable HL gravity and shed some light on the consequences of imposing the projectability condition. Specifically, for the obtained solutions without matter distribution in this paper, the former shows a long range effect like a surplus or deficit solid angle irrespective of the quartic derivative terms but the latter only has a short distance correction like the change of event horizons. In the presence of quartic derivative terms, the effect of those in the former is comparable to the electrostatic field of a point charge but, in some parameter range, the square of it has a negative value.

This paper is organized as follows. In Section II we review HL gravity, setting up our notations. The static spherically symmetric vacuum solutions are obtained in the PG coordinates in Section III, and in the Poincaré coordinates in Section IV. We conclude in Section V.

II. HOŘAVA-LIFSHITZ GRAVITY

The HL gravity has the invariance under the foliation-preserving diffeomorphism $t \rightarrow \tilde{t}(t)$, $x^i \rightarrow \tilde{x}^i(t, \mathbf{x})$, and a scaling behavior in the UV limit $t \rightarrow \ell^3 t$, $x^i \rightarrow \ell x^i$. The action for the HL gravity is best described using the following dynamical variables: the lapse function N , the shift functions N_i , and the three-dimensional spatial metric g_{ij} , with which the metric takes the ADM form

$$ds^2 = -N^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt), \quad (1)$$

where $N^i \equiv g^{ij} N_j$.

In the UV regime there are 5 independent sixth order spatial derivative terms saturating the $z = 3$ anisotropic scaling [5],

$$R^3, \quad R \square R, \quad R_{ij} R^{ij} R, \quad R_{ij} \square R^{ij}, \quad R_{ij} R^i{}_k R^{jk}, \quad (2)$$

which make the theory less predictable. Here R_{ij} is the Ricci tensor of g_{ij} , $R = g^{ij} R_{ij}$, and square(\square) denotes the Laplacian with respect to the spatial metric g_{ij} . Since the detailed balance condition spoils the IR dynamics of HL gravity and the physical motivation for its introduction is not manifest yet, we impose it only in the vicinity of UV fixed point. Then the UV dynamics in the limit of $z = 3$ scaling is assumed to be governed by the quadratic time derivative terms and square of the Cotton tensor [1],

$$C^{ij} \equiv \frac{\epsilon^{ikl} g^{jm}}{\sqrt{g}} \left(R_{lm} - \frac{1}{4} g_{lm} R \right)_{;k}, \quad (3)$$

where ϵ^{ijk} is the antisymmetric tensor density with $\epsilon^{123} = 1$ and semicolon($;$) denotes the spatial covariant derivative. Then the HL gravity action under consideration is given by

$$S_{\text{HL}} = \int dt d^3x N \sqrt{g} \left[\alpha (K_{ij} K^{ij} - \lambda K^2) + \xi R + \sigma \right. \\ \left. + \beta C_{ij} C^{ij} + \gamma \frac{\epsilon^{ijk} g^{lm}}{\sqrt{g}} R_{il} R_{km;j} + \zeta R_{ij} R^{ij} + \eta R^2 \right], \quad (4)$$

where K_{ij} is the extrinsic curvature

$$K_{ij} \equiv \frac{1}{2N} \left(\frac{\partial g_{ij}}{\partial t} - N_{i;j} - N_{j;i} \right). \quad (5)$$

The action (4) possesses 8 parameters, α , λ , ξ , σ , β , γ , ζ , η , and, in the IR limit, α , ξ , and σ terms dominate over higher derivative terms. To recover GR, the renormalization group (RG) running toward the IR limit must lead to

$$\lambda = 1, \quad \alpha = \frac{1}{16\pi G c}, \quad \xi = \frac{c}{16\pi G}, \quad \sigma = -\frac{c\Lambda}{8\pi G}, \quad (6)$$

where c is the speed of light, G is the Newton's constant, and Λ is the cosmological constant.

III. STATIC SPHERICALLY SYMMETRIC SOLUTION IN THE PAINLEVÉ-GULLSTRAND COORDINATES

The projectability condition states that the lapse function is a function of the time only, that is, $N = N(t)$ in the metric (1). Then the time reparametrization, a symmetry transformation in HL gravity, always allows the fixation of $N = 1$. The difficulties without imposing the projectability condition have already been discussed for quantization of HL gravity [7], and then we study the so-called projectable HL gravity in this section. This projectable version is a different theory from non-projectable HL gravity since the foliation-preserving diffeomorphism cannot turn one into the other.

To get the static spherically symmetric vacuum solution under the projectability condition, let us consider the static spherically symmetric metric in the PG coordinates

$$ds^2 = -dt^2 + \frac{1}{f(r)}[dr + n(r)dt]^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (7)$$

which satisfies the projectability condition. The lapse function and the shift function for the metric (1) are unity and n/f , respectively.

The action (4) in terms of f and n is given by

$$S_{\text{HL}} = 4\pi \int dt dr \frac{r^2}{\sqrt{f}} \left\{ -\alpha n^2 \left[\frac{(\lambda - 1)}{4} \left(\frac{2n'}{n} - \frac{f'}{f} \right)^2 + \frac{2\lambda}{r} \left(\frac{2n'}{n} - \frac{f'}{f} \right) + \frac{2(2\lambda - 1)}{r^2} \right] \right. \\ \left. + \frac{(3\zeta + 8\eta)}{2r^2} f'^2 + \frac{2(\zeta + 4\eta)}{r^3} f'(f - 1) + \frac{2(\zeta + 2\eta)}{r^4} (f - 1)^2 - \frac{2\xi}{r^2} (rf' + f - 1) + \sigma \right\}, \quad (8)$$

and the action for the matter-field is

$$S_{\text{M}} = 4\pi \int dt dr \frac{r^2}{\sqrt{f}} \mathcal{L}_{\text{M}}(n, f, \Phi), \quad (9)$$

where Φ stands for all the matter fields of our consideration. Since all the components of Cotton tensor vanish, $C^{ij} = 0$ under this PG metric (7), the contribution from the sixth and fifth order spatial derivative terms disappears in the action and so will do in the equations of motion. It means that the spherically symmetric solutions of our interest are appropriate in describing up to the intermediate energy scale involving quartic spatial derivatives and consistent with our concern on the IR limit and the leading corrections.

The equations of motion are

$$\alpha n^2 \left[(\lambda - 1)r^2 \left(\frac{n''}{n} - \frac{f''}{2f} + \frac{n'^2}{2n^2} - \frac{n'f'}{nf} + \frac{5f'^2}{8f^2} - \frac{f'}{2rf} \right) + 2(2\lambda - 1)r \frac{n'}{n} + 1 \right] \\ + (3\zeta + 8\eta) \left(f''f - \frac{1}{4}f'^2 \right) - \frac{1}{r^2} [(5\zeta + 14\eta)(f - 1)^2 + 2(3\zeta + 8\eta)(f - 1)] - \xi(f - 1) \\ + \frac{\sigma}{2}r^2 = r^2 \left(\frac{1}{f} \frac{\partial \mathcal{L}_{\text{M}}}{\partial f} - \frac{1}{2} \mathcal{L}_{\text{M}} \right), \quad (10)$$

and

$$\alpha(\lambda - 1) \left[r^2 \left(\frac{2n''}{n} - \frac{f''}{f} - \frac{n'f'}{nf} + \frac{f'^2}{f^2} \right) + 4r \frac{n'}{n} - 4 \right] + 2\alpha r \frac{f'}{f} = \frac{1}{n} \frac{\partial \mathcal{L}_{\text{M}}}{\partial n}. \quad (11)$$

Here the equations of motion have been obtained by directly inserting the metric ansatz (7) into the action and then by variation of the action. Since the metric has good symmetries, (10)–(11) coincide with the Euler-Lagrange equations for static spherically symmetric objects, derived from the action (4) after assigning the projectability condition $N = 1$.

Now we consider the vacuum solution for which $\mathcal{L}_M = 0$. Concerning the character of the solution, we consider the $\lambda = 1$ case for tractability, which is also consistent with the recovery of GR in the IR limit. A newly proposed version of HL gravity, which is free from the unwanted scalar graviton, possesses an extra local U(1) symmetry to the foliation-preserving diffeomorphism naturally fixes λ to be unity [2]. With $\lambda = 1$ the equation (11) is reduced to $f' = 0$ so that we write a constant solution

$$f(r) = 1 + f_0 > 0. \quad (12)$$

Inserting (12) into the equation (10), we obtain

$$(rn^2)' = -\frac{\sigma}{2\alpha}r^2 + \frac{\xi}{\alpha}f_0 + \frac{(5\zeta + 14\eta)f_0^2 + 2(3\zeta + 8\eta)f_0}{\alpha r^2}. \quad (13)$$

The solution to this equation is

$$n(r) = \pm \sqrt{-\frac{\sigma}{6\alpha}r^2 + \frac{\xi}{\alpha}f_0 + \frac{r_s}{r} - \frac{(5\zeta + 14\eta)f_0^2 + 2(3\zeta + 8\eta)f_0}{\alpha r^2}}, \quad (14)$$

where r_s is the integration constant which can be identified as $r_s = 2GM$ and M is the mass of black hole as in the Schwarzschild solution. Two comments are noted before analyzing the solution. First, if $f_0 = 0$, the solution reduces to that of GR even for $\xi/\alpha \neq 1$ and in the presence of the higher spatial derivative terms. Second, the full set of field equations of the non-projectable version of HL gravity up to quartic spatial derivative terms were derived in [8, 11]. The projectability condition $N = N(t)$ is not compatible with those equations in general. Since the non-projectable version is a different theory from the projectable version of our interest, our analysis will be focused on the obtained solution (14).

In the GR limit where $\xi/\alpha = 1$ and $\sigma = \zeta = \eta = 0$, f_0 can be rescaled to be zero by an appropriate t - r mixing coordinate transformation. On the other hand, in HL gravity, such t - r mixing coordinate transformation is not allowed as a symmetry transformation. Thus, we have an additional integration constant f_0 . Let us examine the implications of this new integration constant. With $\sigma = 0$, the solution is given by

$$f(r) = 1 + f_0, \quad n(r) = \pm \sqrt{\frac{\xi}{\alpha}f_0 + \frac{r_s}{r} - \frac{d}{r^2}}, \quad (15)$$

where $d = [(5\zeta + 14\eta)f_0^2 + 2(3\zeta + 8\eta)f_0]/\alpha$. When $\xi/\alpha = 1$, it looks the same as the Reissner-Nordström solution in the PG coordinates with the identification $d = Gq^2$ where q is the electric charge. When $f_0 \neq 0$, the $1/r^2$ -term acts like that of an electric charge. The important difference from the Reissner-Nordström solution of GR is that the coefficient of $1/r^2$ -term can have a positive value when ζ or $\eta < 0$. Though the quadratic curvature terms induce possibility of $d = Gq^2 > 0$, we will read the meaning of it in the context of GR. If we write down the Einstein equation, $G^\mu_\nu = -8\pi GT^\mu_\nu$, in terms of the metric (7) with (12), it becomes

$$(rn^2)' = f_0 - 8\pi GT^t_t r^2 = f_0 + 4\pi G f_0 E_r^2 r^2, \quad (16)$$

where the last equality holds for the radial component of electrostatic field $E_r = F_{rt}$. Comparing (16) with (13) in the limit of $\sigma = r_s = \xi/\alpha - 1 = 0$, we identify the energy density as

$$-T_t^t = \frac{f_0}{2} E_r^2 = \frac{d}{8\pi G r^4}. \quad (17)$$

Therefore, in order to obtain the same solution $n(r)$ (15) in GR, we need $E_r^2 = q^2/4\pi f_0 r^4$ which can be negative for $d = Gq^2 < 0$. In the context of GR, it is forbidden since this matter configuration violates the positive energy theorem. In conventional electromagnetism the effect of $q^2 < 0$ may imply sign-flipped Coulomb force, attractive between same charges. However, in HL gravity, the solution (15) with negative d is generic. It is obtained in the absence of matter, $\mathcal{L}_M = 0$, but in the presence of quartic spatial derivative terms, the third term in the right-hand side of the equation (13). Since violation of the positive energy theorem and modification of such Coulomb force are unphysical, detection of the signal of $-d/r^2 > 0$ term suggests existence of the era of HL gravity.

If f_0 cannot be scaled to zero in the metric of (15), another intriguing question is to address the effect of the constant term $\xi f_0/\alpha$ which is unphysical in GR and is not related to higher spatial derivatives. To see this explicitly, let us examine the geodesic equation of a test body. Let the constants of motion corresponding to the cyclic coordinates t and ϕ be E and ℓ , respectively. Then radial geodesic equation can be written as

$$\dot{r}^2 + \left(1 + f_0\delta - \frac{r_s}{r} + \frac{d}{r^2}\right) \left(1 + \frac{\ell^2}{r^2}\right) = E^2, \quad (18)$$

where the overdot denotes differentiation with respect to the proper time of the test body. We introduced the factor $\delta = 1 - \xi/\alpha$ measuring the deviation of the propagation speed from unity. From this equation, we obtain the orbit equation for $u(\phi) \equiv 1/r(\phi)$

$$\frac{d^2 u}{d\phi^2} + \left(1 + f_0\delta + \frac{d}{\ell^2}\right) u = \frac{r_s}{2\ell^2} + \frac{3r_s}{2} u^2 + 2d u^3. \quad (19)$$

When we turn off the higher derivative terms $d = 0$ and the mass $r_s = 0$, the orbit equation reduces to a linear equation and the coefficient of u -term decides the allowed range of the angle ϕ . Solution of it is $u(\phi) \propto \cos(\sqrt{1 + f_0\delta} \phi)$, and thus, non-zero f_0 leads to the surplus solid angle $\Delta = 2\pi(\sqrt{1 + f_0\delta} - 1)$ for $f_0\delta > 0$ and the deficit solid angle $\Delta = 2\pi(1 - \sqrt{1 + f_0\delta})$ for $-1 < f_0\delta < 0$. When $f_0\delta = 0$, the space has neither surplus nor deficit solid angles.

In the context of GR (16), the constant metric solution, $n = \xi f_0/\alpha$, from (13) can be obtained by assuming the energy density, $-T_t^t = -f_0\delta/8\pi G r^2$. For the deficit solid angle with $-1 < f_0\delta < 0$, the energy density is positive and it corresponds to gravitating global monopole [14]. For the surplus solid angle with $f_0\delta > 0$, the energy density is negatively distributed everywhere. It is forbidden in GR since it violates the positive energy theorem. However, in HL gravity, this constant solution is a static solution attained in the absence of matter field, $\mathcal{L}_M = 0$, and higher derivatives, $\zeta = \eta = 0$. This result is also contrasted with the case of HL gravity with the detailed balance condition, in which a positive energy distribution of the electrostatic field of a point charge supports the geometry involving surplus solid angle [15].

Possible astrophysical effects of a deficit/surplus solid angle can easily be visualized in the case of $\theta = \pi/2$, where an observer, a light source, and the the apex of deficit/surplus

solid angle are in the same plane. When a geometry with deficit solid angle is formed, the light from a star behind the apex propagates straight and arrives at a static observer who detects double images projected behind the source [16]. When a geometry with surplus solid angle is formed, a static observer tracking down the trajectory of the star experiences sudden disappearance of its image for the period proportional to the surplus angle and reappearance at a distant point over the apex of surplus angle [15]. The present astronomical bound of angular resolution is about 200 micro-arcseconds ($\sim 10^{-9}$ radian) [17]. It means that, in principle, it can be observed if the graviton speed deviates away from unity satisfies $|\delta| = |1 - \xi/\alpha| > 10^{-9}\pi/f_0$. Though f_0 is an undetermined free parameter in the present stage, the formula suggests $10^{-8} \sim 10^{-9}$ deviation of the graviton speed for $f_0 \sim \mathcal{O}(1)$, which looks extremely stringent for the RG flows for the ratio ξ/α in the vicinity of IR fixed point.

IV. STATIC SPHERICALLY SYMMETRIC SOLUTION IN THE POINCARÉ COORDINATES

In this section, we consider static spherically symmetric vacuum solution in non-projectable HL gravity for comparison. This is done by taking a static spherically symmetric metric in the Poincaré coordinates

$$ds^2 = -N(r)^2 d\tilde{t}^2 + \frac{1}{F(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi)^2. \quad (20)$$

The metric (7) is connected to the metric (20) by a coordinate transformation

$$\tilde{t} = \sqrt{C} t + \int \frac{\sqrt{f-F}}{F} dr, \quad (21)$$

with r, θ, ϕ unchanged and

$$f = C \frac{F}{N^2}, \quad n^2 = (C - N^2) \frac{F}{N^2}, \quad (22)$$

where C is a constant. Since this transformation is not a foliation-preserving diffeomorphism, the solutions found using the metric (20) and the metric (7) will not be equivalent in HL gravity.

The action (4) written in terms of N and F is

$$S_{\text{HL}} = 4\pi \int dt dr \frac{r^2 N}{\sqrt{F}} \left[\frac{3\zeta + 8\eta}{2r^2} F'^2 + \frac{2(\zeta + 4\eta)}{r^3} F'(F-1) + \frac{2(\zeta + 2\eta)}{r^4} (F-1)^2 - \frac{2\xi}{r^2} (rF' + F-1) + \sigma \right], \quad (23)$$

and the matter-field action with spherical symmetry is

$$S_{\text{M}} = 4\pi \int dt dr \frac{r^2 N}{\sqrt{F}} \mathcal{L}_{\text{M}}(N, F, \Phi). \quad (24)$$

The equations of motion obtained from the variation of N and F are

$$\frac{3\zeta + 8\eta}{2r^2}F'^2 + \frac{2(\zeta + 4\eta)}{r^3}F'(F - 1) + \frac{2(\zeta + 2\eta)}{r^4}(F - 1)^2 - \frac{2\xi}{r^2}(rF' + F - 1) + \sigma = -\mathcal{L}_M - N\frac{\partial\mathcal{L}_M}{\partial N}, \quad (25)$$

$$\left(\log \frac{N}{\sqrt{F}}\right)' \left[\frac{3\zeta + 8\eta}{r^2}F' + \frac{2(\zeta + 4\eta)}{r^3}(F - 1) - \frac{2\xi}{r} \right] + \frac{(3\zeta + 8\eta)}{r^2} \left[F'' - \frac{2}{r^2}(F - 1) \right] = \frac{\partial\mathcal{L}_M}{\partial F} + \frac{N}{F} \frac{\partial\mathcal{L}_M}{\partial N}, \quad (26)$$

where the prime denotes the derivative with respect to r . Now we consider the vacuum solution of (25)–(26) for which $\mathcal{L}_M = 0$. The first equation involves $F(r)$ only and the second equation determines $N(r)$ from $F(r)$ obtained from the first equation. Note that both equations are independent of α , β , and γ since the extrinsic curvature and the Cotton tensor vanish for the static spherically symmetric metric (20). When compared to (10) and (11), a notable difference is that (25) and (26) are independent of λ .

We write the metric function F as

$$F(r) = 1 + r^2[a - p(r)], \quad (27)$$

where

$$a = \frac{\xi - \tilde{\xi}}{4(\zeta + 3\eta)}, \quad \tilde{\xi} = \sqrt{\xi^2 - \frac{4}{3}\sigma(\zeta + 3\eta)}. \quad (28)$$

Then, from (25), $p(r)$ satisfies the equation

$$(3\zeta + 8\eta)r^2p'^2 + 4[4(\zeta + 3\eta)p + \tilde{\xi}]rp' + 12[2(\zeta + 3\eta)p^2 + \tilde{\xi}p] = 0. \quad (29)$$

When both ζ and η vanish, the well-known GR solution

$$N^2(r) = F(r) = 1 - \frac{\sigma}{6\xi}r^2 - \frac{r_s}{r} \quad (30)$$

is obtained, regardless of the coefficient of scalar curvature term, ξ . Here r_s is the same integration constant as in (14), $r_s = 2GM$. Note that when we have nonvanishing cosmological constant, $\sigma \neq 0$, the asymptotic behavior is determined by the coefficient of r^2 term. When higher derivative terms are introduced, its coefficient is modified and hence the asymptotic behavior becomes different from that of GR, irrespective of the assignment of the detailed balance condition.

When the combination $3\zeta + 8\eta$ vanishes, the equation (29) is easily integrated and the solution is given by

$$N^2(r) = F(r) = 1 + ar^2 - \frac{2r^2}{\tilde{\zeta}} \left(1 - \sqrt{1 - \frac{\tilde{\zeta}r_s}{r^3}} \right), \quad (31)$$

where $\tilde{\zeta} \equiv \zeta/\xi$. Let us consider the case $\sigma = 0$. Then, we have $a = 0$ and $\tilde{\xi} = \xi$. For large r ($r \gg (\tilde{\zeta}r_s)^{1/3}$), we get an approximation

$$N^2(r) = F(r) \approx 1 - \frac{r_s}{r} - \frac{\tilde{\zeta}r_s^2}{4r^4}, \quad (32)$$

which shows the usual behavior of the Schwarzschild black hole plus small corrections due to the higher derivative terms. On the other hand, at short distance, the position of event horizon is modified. When $\tilde{\zeta} < 0$, we have two event horizons at

$$r_H = \frac{1}{2} \left[r_s \pm \sqrt{r_s^2 - (-\tilde{\zeta})} \right]. \quad (33)$$

When $\tilde{\zeta} > 0$, we have an event horizon at

$$r_H = \frac{1}{2} \left(r_s + \sqrt{r_s^2 + \tilde{\zeta}} \right). \quad (34)$$

For $3\zeta + 8\eta \neq 0$, we solve (29) for $p'(r)$ and obtain

$$\tilde{p}' = -\frac{6}{(1+8b)r} \left(1 + 4b\tilde{p} - \sqrt{1 - \tilde{p} - 2b\tilde{p}^2} \right), \quad (35)$$

where $\tilde{p} = \tilde{\zeta}p$ and $b = 1 + 3\eta/\zeta$. When $b = 0$, this equation is easily integrated to give

$$(\sqrt{1 - \tilde{p}} - 1)e^{(\sqrt{1 - \tilde{p}} - 1)} = -\frac{\tilde{\zeta}r_s}{2r^3}. \quad (36)$$

For $b \neq 0$, we get

$$\frac{\tilde{p}^2 \left[1 + 2b \left(2 + \tilde{p} + 2\sqrt{1 - \tilde{p} - 2b\tilde{p}^2} \right) \right] \left(\frac{1+4b\tilde{p}}{\sqrt{-2b}} + 2\sqrt{1 - \tilde{p} - 2b\tilde{p}^2} \right)^{\frac{1}{\sqrt{-2b}}}}{2 - \tilde{p} + 2\sqrt{1 - \tilde{p} - 2b\tilde{p}^2}} = \frac{c}{r^6}, \quad (37)$$

where c is a constant. To get $\tilde{p}(r)$ explicitly, we need to invert these equations. Assuming \tilde{p} is small, we can solve the equation perturbatively and obtain

$$\tilde{p} \approx -\frac{\tilde{\zeta}r_s}{r^3} \left(1 - \frac{\tilde{\zeta}r_s}{4r^3} \right)^{-1}. \quad (38)$$

Then up to this order

$$F(r) = 1 + ar^2 - \frac{r_s}{r} - \frac{\zeta r_s^2}{4\tilde{\xi}r^4}, \quad (39)$$

$$N^2(r) = F(r) + \frac{3a\zeta(3\zeta + 8\eta)r_s^2}{4\tilde{\xi}^2r^4}. \quad (40)$$

In general, the higher derivative terms give rise to the subleading corrections of order $\zeta r_s/\xi r^3$. Thus, Eddington-Robertson parameters are same as those of GR, and it is hard to observe the macroscopic effect of them. However, higher spatial derivative terms modify the causal structure at short distance.

In the previous and present sections, we obtain static vacuum solution of a projectable and a non-projectable version of HL gravity without the detailed balance condition at the IR regime, which are differentiated by two inequivalent metrics. For vanishing cosmological constant case, static spherically symmetric solution for the metric without nontrivial lapse function in the PG coordinates supports geometry of a surplus or deficit solid angle, however the other solution in the Poincaré coordinates does not. Instead of such a long range effect, it involves a short distance correction like the change of event horizons.

V. CONCLUSION

We obtained static spherically symmetric vacuum solutions of the recently proposed HL gravity, imposing the detailed balance condition only in the UV limit. Since HL gravity is intrinsically non-relativistic, the choice of a preferred frame is unavoidable. The choice of coordinate system is related to the choice of the frame. For static spherically symmetric vacuum solutions, we tried two coordinate systems, the PG coordinates and the Poincaré coordinates. They are connected by the time-space mixing coordinate transformation which is not foliation-preserving. Thus, the solutions in two coordinate systems are distinct and physically inequivalent. The distinguishing feature of metrics in two coordinate systems is that the one in PG coordinates satisfies the projectability condition and the other does not.

Even in the absence of higher derivative terms, the solution without matter distribution in the PG coordinates has an additional integration constant which leads to the geometry of a surplus or deficit solid angle. This long range effect can be constrained by smallness of the astronomical bound of angular resolution, and so do some parameters of HL gravity. In both solutions, the effect of higher spatial derivative terms is that they change the causal structures at short distances. But the differences from GR are not observationally significant in the macroscopic world for the reasonable choice of parameters. This is as expected because the higher spatial derivative terms are in general suppressed by the Planck scale or, if any, the new quantum gravity scale which may be smaller than the Planck scale but still out of our experimental reach.

The solution in PG coordinate system may be stated as the static spherically symmetric vacuum solution of projectable HL gravity. For this solution we found that a new integration constant must be introduced in addition to mass, when ξ/α differs from unity or we have non-vanishing higher spatial derivative terms. This constant implies the existence of surplus or deficit angle for the former and the electric charge for the latter. The current astronomical bound on surplus/deficit angle can impose stringent constraint on the deviation of ξ/α from unity. One striking feature of the solution is that the electric charge can effectively be imaginary depending on the signature of the couplings of higher spatial derivative terms.

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